Controlling chaotic frictional forces

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Methods to control friction in systems under shear are proposed, which enable eliminate chaotic stick-slip motion, and modify frictional forces. The possibilities to control friction are demonstrated using a model system that displays the main experimentally observed behaviors, obtained in measurements on nanoscale confined liquids and granular layers. The methods should be applicable to real systems for which time series of dynamical variables are experimentally available. The only necessary condition is the existence of (possibly unstable) sliding regimes of motion in the experimental systems. [S1063-651X(98)06606-9]

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Recent experiments allow for detailed investigations of frictional forces of thin liquids sheared between two solid surfaces [1–3], or of sheared granular layers [4]. Summarizing the experimental observations, one distinguishes between a low driving velocity region, where the system exhibits dry-friction-like behavior, and higher driving velocities, which correspond to a more lubricated-like behavior. The low velocity regime is characterized by chaotic stick-slip motion, which is basically determined by the interplay between static and kinetic friction forces, and whose details depend on the mechanical properties of the probing system [2,5,6]. For high velocities the system displays smooth sliding, which resembles thinning of the effective viscosity. Low- and high-velocity regimes are separated by a well defined critical driving velocity v_c.

From a practical point of view one wishes to be able to control frictional forces so that the overall friction is reduced or enhanced, the chaotic regime is eliminated, and instead smooth sliding is achieved. Such control can be of high technological importance for micromechanical devices, for instance, in computer disk drives, where the early stages of motion and the stopping process, which exhibit chaotic stick slip, pose a real problem [7].

Controlling frictional forces has been traditionally approached by chemical means, namely, using lubricating liquids. A different approach, proposed here, is by controlling the system mechanically. Our goal is twofold: (a) to achieve smooth sliding at low driving velocities, which otherwise correspond to the stick-slip regime; (b) to decrease the frictional forces.

In this paper we demonstrate the possibility to control friction in a model that has been shown to display the main experimentally observed properties obtained in measurements on nanoscale confined liquids [1–3], and granular layers [4]. Although we discuss here a specific simple model, the possibility to control friction mechanically, and the methods applied for the control, are model independent and amenable to experimental verification.

We consider a one-dimensional model that includes two rigid plates and a single particle of mass m embedded between them. The interaction between the particle and each of the plates is described by a periodic potential $U_p(x)$. There is no direct interaction between the plates. The top plate of mass M is pulled by a linear spring with a force constant K connected to a stage that moves with a velocity v (see Fig. 1 for a sketch of the model). Experimentally one usually follows the time dependence of the spring force for a fixed, or varying in time, stage velocity [1–3].

Before analyzing the model and describing the control methods, we present in Fig. 2 an example of a chaotic stick-slip behavior typical to the low driving velocities. Lines (2) and (3) in Fig. 2 correspond to different types of sliding states [8]. These states are unstable for low driving velocities but, as shown below, could be stabilized by the control methods proposed here. The main idea of this work is to replace chaotic stick-slip motion (line 1) by smooth sliding (lines 2 or 3) in the low velocity region. Figure 3 displays the result of a control, applied in the time window $t_1<t<t_2$, where smooth sliding is achieved. Figure 4 illustrates another method of control. The control is switched on in the high velocity range, which corresponds to stable sliding, and then the controllable system is decelerated toward the low velocity region keeping the chosen sliding state. As a characteristic force unit in Figs. 1–4 we use the value of the static frictional force F_s [8]. Figures 3 and 4 clearly demonstrate the possibility to drive chaotic motion to smooth sliding and to decrease the frictional force by controlling the system mechanically.

We now turn to the description of the model and of the

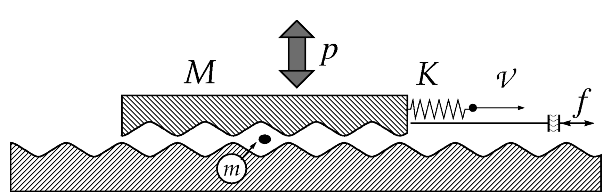

FIG. 1. Schematic sketch of a model geometry.

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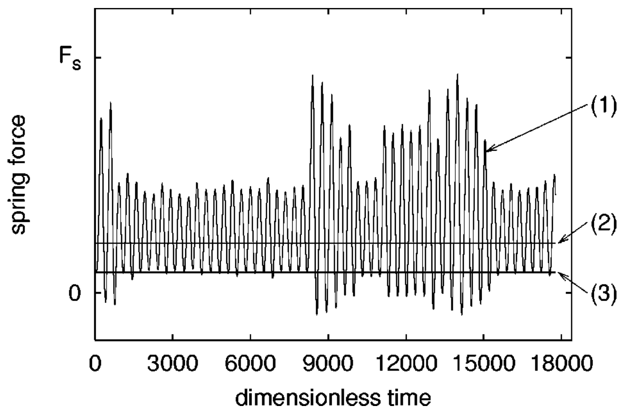

FIG. 2. Typical time series of the spring force for dimensionless driving velocity $v=0.27$: (1) chaotic stick-slip motion without control, (2) trapped-sliding state stabilized by control, (3) decoupled sliding state stabilized by control. Spring force is presented in units of static friction force, $F_s=2\pi U_0/b$ [8].

details of the control methods. The coupled equations of motion for the top plate and the particle are

$$M\ddot{X}+\eta(\dot{X}-\dot{x})+K(X-\mathcal{V}t)+\frac{\partial U_p(x-X)}{\partial X}=f(t), \quad (1a)$$

$$m\ddot{x}+\eta\dot{x}+\eta(\dot{x}-\dot{X})+\frac{\partial U_p(x)}{\partial x}+\frac{\partial U_p(x-X)}{\partial x}=0, \quad (1b)$$

where x and X are the coordinates of the particle and the top plate, respectively. The second term in Eq. (1a) and the second and the third terms in Eq. (1b) describe the dissipative forces between the particle and the plates and are proportional to their relative velocities. These terms account for dissipation due to phonons and/or other excitations. The third term in Eq. (1a) is the driving force due to the stage motion. The additional pulling force $f(t)$ is introduced here in order to shorten the transient time for switching of the control, and will be discussed later. The remaining terms are due to the periodic interaction potential between the particle and the plates.

We choose the potential $U_p(x)$ to be

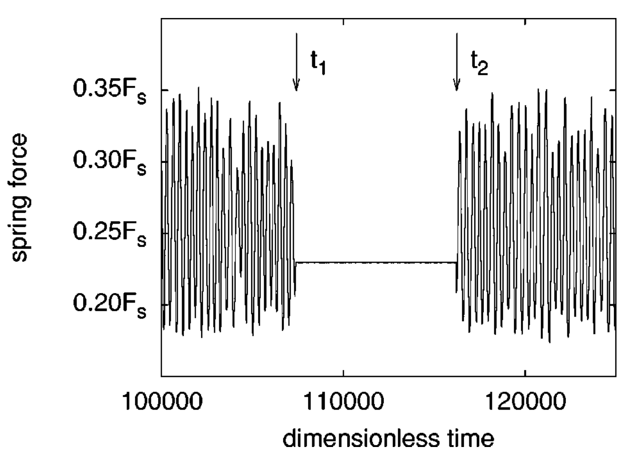

FIG. 3. Eliminating the chaotic stick-slip motion under mechanical control. Dimensionless driving velocity $v=0.34$. Spring force is presented in units of static friction force, $F_s=2\pi U_0/b$ [8].

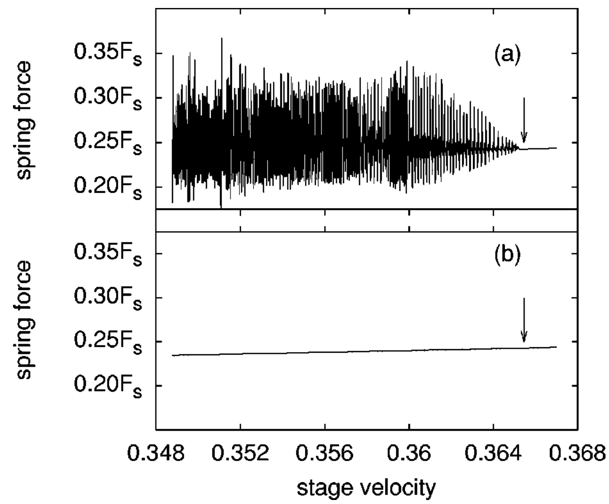

FIG. 4. Time oscillations of the spring force for the deceleration of the driving stage: (a) without control, (b) under control. Spring force is presented in units of static friction force, $F_s=2\pi U_0/b$ [8]. For convenience the stage velocity (instead of time) is indicated on the axis. Vertical arrows indicate the critical velocity v_c.

$$U_p(x)=-U(p)\cos\left(\frac{2\pi}{b}x\right). \quad (2)$$

The amplitude of the periodic potential that the particle feels depends on the normal load which we use as the control parameter:

$$U(p)=U_0[1+\beta(p-p_0)]. \quad (3)$$

U_0 is the value of the potential for some nominal value of the normal load p_0, and β is a dimensional constant. Equation (3) assumes small load variations around p_0, which, as shown below, are sufficient to achieve control.

The dynamical behavior of the model is determined by the following dimensionless parameters: $\gamma=\eta/(m\omega)$ is a dimensionless friction constant, where $\omega=(2\pi/b)\sqrt{U_0/m}$ is the frequency of the oscillations of the particle in the minima of potential; $\epsilon=m/M$ is the ratio of particle and plate masses; $\alpha=\Omega/\omega$ is the ratio of frequencies of the free oscillations of the top plate $\Omega=\sqrt{K/M}$ and the particle; and $v=\mathcal{V}/(\omega b)$ is the dimensionless stage velocity. In the calculations reported in this paper we use parameter values that belong to the underdamped [5] case: $\alpha=0.02$, $\gamma=0.1$, and $\epsilon=0.125$. The control methods should apply to the overdamped limit as well.

For $f=0$ and $p=p_0$ the model presented on Fig. 1 leads to a number of dynamical behaviors as the stage velocity is varied. We have observed four quantitatively different dynamical regimes [8,9]: (a) stick-slip motion of the top plate at low velocities; (b) as the stage velocity increases the motion of the top plate is characterized by irregular stop events with time intervals between them that increase rapidly with v; here the stick-slip motion becomes erratic; (c) in the kinetic regime the top plate never stops and the spring executes chaotic oscillations, and (d) two types of smooth sliding occur when the stage velocity is above the critical velocity v_c.

With the exception of very small driving velocities ($v<v_0$, where $v_0=0.03$ for the chosen numerical values of

system parameters) the dynamics of the system is chaotic for the velocity regimes (a)–(c). Namely, the largest Liapunov exponent, which provides a quantitative measure of the degree of stochasticity, is positive [8,9]. The velocity dependence of the Liapunov exponent gives a clear manifestation of the transition to sliding [8]. As the stage velocity increases and approaches a critical velocity v_c, the largest Liapunov exponent decreases steeply and becomes negative at $v = v_c$, suggesting the disappearance of chaos in the transition and the onset of the sliding regime (d). This concurs with the decrease in the amplitude of the spring force oscillations. The value of the critical velocity for our chosen values of parameters is $v_c = 0.365$.

For driving velocities v in the range $v_c < v < v_c^t$, where $v_c^t \approx 1.59$, the system is in the sliding regime and the particle is trapped by one of the plates and performs small oscillations around a minimum of the particle-plate interaction potential. The frictional force corresponding to this *trapped-sliding state* is [8]

$$F(\dot{x}) \approx \eta \dot{x}. \tag{4}$$

At stage velocities $v > v_c^u$, where $v_c^u \approx 0.59$, the character of sliding changes. The particle ceases to feel the corrugation of the plates and moves with the velocity $v/2$. This transition to the *decoupled-sliding state* is accompanied by a drop in the frictional force, which becomes the same as for flat plates [8],

$$F(\dot{x}) \approx \tfrac{1}{2} \eta \dot{x}. \tag{5}$$

It should be noted that Eq. (5) is the lower bound for the frictional force at a given driving velocity. If the stability intervals of trapped and decoupled states overlap, i.e., $v_c^{(t)} > v_c^{(u)}$, as for our choice of parameters, there is a bistability region $v_c^u < v < v_c^t$, and we have hysteretic behavior.

Our aim is to stabilize the sliding states for driving velocities $v < v_c$, where one would expect chaotic stick-slip motion. Sliding states correspond to stable periodic orbits of the system with two periods: (a) period $T = 1/v$, which corresponds to a motion of the particle being trapped by one of the plate; (b) period $T = 2/v$, which corresponds to the particle moving with the drift velocity $v/2$. In the chaotic region both orbits still exist, but are unstable. Our approach is therefore to drive the system into a sliding state by controlling these unstable periodic orbits (UPO). This makes it possible to extend the smooth sliding to lower velocities. The control of such orbits in dynamical systems have been proposed [10] and experimentally applied to a wide variety of physical systems including mechanical systems, lasers, semiconductor circuits, chemical reactions, biological systems, etc. (see [11] for references).

We are interested in controlling the chaotic friction at small velocities and also in maintaining smooth sliding when starting at $v > v_c$ and decelerating the system. Here we present the results for the control of the trapped-sliding state. Figure 3 demonstrates the effect of the mechanical control on the time dependence of the spring force in the kinetic regime (c), $v = 0.34$. The control is switched on at time t_1 and is shut down at time t_2. We clearly see that as a result of the control the chaotic motion of the top plate is replaced by a smooth sliding, which corresponds to a trapped state with the frictional force given by Eq. (4). Figure 4 illustrates another method of catching the desired orbit, where we start from high driving velocities $v > v_c$, for which the trapped-sliding state is stable. The system then decelerates under control, keeping the chosen sliding state until we reach the velocity of interest. Figures 4(a) and 4(b) exhibit the velocity dependence of the spring force without and under control, correspondingly. The control of the decoupled-sliding state performed with the help of the second approach will be discussed in a separate publication. The use of this state gives an additional possibility to decrease friction (twice compared to the trapped state).

We now outline the methods that we use to obtain the results. Control methods are characterized by two independent steps: (a) reaching the vicinity of an UPO and (b) stabilizing the system. First we present the chosen variant of the stabilizing of an UPO. Our model system is described by the continuous-time nonautonomous system of differential equations $d\mathbf{X}/dt = F(\mathbf{X}, t, p)$, where $\mathbf{X}$ is a four-dimensional state vector $(x, \dot{x}, X, \dot{X})$. For simplicity we assume that the system depends on a single control parameter p, which in our case is normal load, that can be externally adjusted. Let us measure the state of the system at times $t_n = nT$, where T is the period of a desired orbit. Denoting $\mathbf{X}_n = \mathbf{X}(t_n)$, we have a discrete-time mapping $\mathbf{X}_{n+1} = \Phi(\mathbf{X}_n, p)$. We restrict the variations of the control parameter p to some small interval near a nominal value p_0. For a periodic orbit we have $\mathbf{X}_n = \mathbf{X}_{n+1} = \mathbf{X}_{n+2} = \cdots \equiv \mathbf{X}_*(p_0)$, i.e., the periodic orbit of a system of differential equations is a fixed point of the corresponding mapping. Let us assume that the dynamical system is in a close neighborhood of the periodic orbit. Denote the deviation of the trajectory from the chosen periodic orbit by $\delta\mathbf{X}_n = \mathbf{X}_n - \mathbf{X}_*(p_0)$, and by δp the deviation of the control parameter from its nominal value, $\delta p = p_n - p_0$. Linearizing the mapping equations we get $\delta\mathbf{X}_{n+1} = \mathsf{A}\delta\mathbf{X}_n + \mathbf{B}\delta p$. Here A is a Jacobian matrix and $\mathbf{B}$ is a column vector. Their determination in our case is possible only numerically (the corresponding methods are well developed; see, for example, in [12], Chap. 4). For an unstable periodic orbit at least one of the eigenvalues of A is greater than unity in magnitude. Our goal is to stabilize the chosen orbit by varying the control parameter. As a control mechanism we chose a version of proportional feedback, i.e., $\delta p_n = -\mathbf{K}^T \cdot \delta\mathbf{X}_n$, where the column vector $\mathbf{K}$ is to be determined, so that the fixed point $\mathbf{X}_*(p_0)$ becomes stable. With the proportional feedback control we have for the deviations $\delta\mathbf{X}_{n+1} = (\mathsf{A} - \mathbf{B} \otimes \mathbf{K}^T)\delta\mathbf{X}_n$, which shows that the periodic orbit will be stable if all eigenvalues of the matrix $(\mathsf{A} - \mathbf{B} \otimes \mathbf{K}^T)$ have moduli smaller than unity. The solution to the problem of the determination of $\mathbf{K}$, such that the eigenvalues of the matrix $(\mathsf{A} - \mathbf{B} \otimes \mathbf{K}^T)$ have *arbitrary* specified values, is well known from control theory (see [13] and references therein). In particular in our numerical calculations we chose $\mathbf{K}$ such that (a) all eigenvalues of the matrix $(\mathsf{A} - \mathbf{B} \otimes \mathbf{K}^T)$ are zero, or (b) only unstable eigenvalue of the matrix $(\mathsf{A} - \mathbf{B} \otimes \mathbf{K}^T)$ are zero.

The control mechanism described above can be applied when a trajectory of the system falls into a sufficiently small region around desired UPO. Two methods of ''forcing'' the system to reach a close neighborhood of these orbits are used in this work. The first method, proposed in [10], is applicable for control of any *chaotic system* and could be applied to control friction in the range of velocities $v_0 < v < v_c$. It relies on the ergodic wandering of the trajectory to bring it close to the desired state. Here chaos actually is advantageous. For chaotic systems typical times required to approach a small region are of order of ξ^{-D}, where ξ is the linear dimension of the region and D is the dimension of the phase space. For relatively high dimensional systems, such as more realistic models of friction, the transient times may be extremely long. In the present work, in order to shorten the transient time for control switching, we use the following trick. We apply an additional pulling force (force f in Fig. 1) that effectively reduces the dimension of system from 4 to 3. The force is applied when three of four dynamical variables, namely, the position and the velocity of the embedded particle and the velocity of the top plate, are in a small region around their values for a periodic orbit. The deviation of the position of the top plate from the value on the periodic orbit, or, what is equivalent, the deviation of the length of the pulling spring, is compensated by additional force, $f(t)$. This dimensional reduction is sufficient to decrease the transient time to an acceptable value. The results, demonstrating controlled motion of the system are presented in Fig. 3. The time scale for control is $b/\mathcal{V}$ in our model, but since we deal with very low stage velocities the value is reasonable. In addition, if the system is moderately chaotic the control could be applied every N periods. In our case $N \sim 100$.

The second method is more specific to the *problem of friction*, but it is easily applicable for systems of any dimension. The method utilizes the fact that desired periodic orbits are stable for higher stage velocities, namely, in the sliding regimes. The range of stability is different for each orbit. Therefore, we start from driving velocities for which the desired periodic orbit is stable. Then we decelerate the stage gradually in small velocity steps, until we reach the velocity of interest. During the decelerating process the control is permanently switched on so that the system remains on the periodic orbit. The results, demonstrating controlled motion of the system by this method, are presented on Fig. 4.

In the present work we have demonstrated the possibility to control friction in a model system described by differential equations. For realistic systems time series of dynamical variables, rather than governing equations, are experimentally available. In this case the time-delay embedding method [14] could be applied in order to transform a scalar time series into a trajectory in phase space. This procedure allows one to find the desired unstable periodic orbits and to calculate variations of parameters required to control friction.

In conclusion, methods to control friction in a model system have been proposed to avoid the chaotic behavior, already at low velocities, and to achieve the lower bound of the friction force. The choice of control parameters is not unique and practically any system parameter or combination of system parameters can act as a control parameter. The only necessary condition for the application of the proposed methods is the existence of unstable orbits corresponding to sliding regimes of motion.

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[1] H. Yoshizawa, P. McGuiggan, and J. Israelachvili, Science **259**, 1305 (1993).
[2] A. L. Demirel and S. Granick, Phys. Rev. Lett. **77**, 4330 (1996).
[3] J. Klein and E. Kumacheva, Science **269**, 816 (1995).
[4] S. Nasuno, A. Kudrolli, and J. P. Gollub, Phys. Rev. Lett. **79**, 949 (1997).
[5] M. G. Rozman, M. Urbakh, and J. Klafter, Europhys. Lett. **39**, 183 (1997).
[6] A. D. Berman, W. A. Ducker, and J. N. Israelachvili, Langmuir **12**, 4559 (1996).
[7] C. M. Mate and A. M. Homola, in *Micro/Nanotribology and Its Applications*, Vol. 330 of *NATO Advanced Sciences Institutes Series E: Applied Sciences*, edited by B. Bhushan (Kluwer Academic Publishers, Dordtrecht, 1997), pp. 647–661.
[8] M. G. Rozman, M. Urbakh, and J. Klafter, Phys. Rev. E **54**, 6485 (1996).
[9] M. G. Rozman, M. Urbakh, and J. Klafter, Phys. Rev. Lett. **77**, 683 (1996).
[10] E. Ott, C. Grebogi, and J. A. Yorke, Phys. Rev. Lett. **64**, 1196 (1990).
[11] E. Barreto *et al.*, Phys. Rev. E **51**, 4169 (1995).
[12] P. Cvitanović (unpublished), available from http://www.nbi.dk/~predrag/QCcourse.
[13] K. Ogata, *Discrete-time Control Systems* (Prentice-Hall, Englewood Cliffs, NJ, 1995).
[14] H. D. I. Abarbanel, R. Brown, J. L. Sidorowich, and L. S. Tsimring, Rev. Mod. Phys. **65**, 1331 (1993).